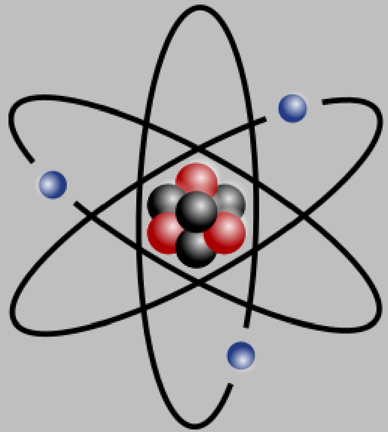
International House Tashkent
Subject: Physics
Department: ES, Course 1
Lesson 10. Simple harmonic motion

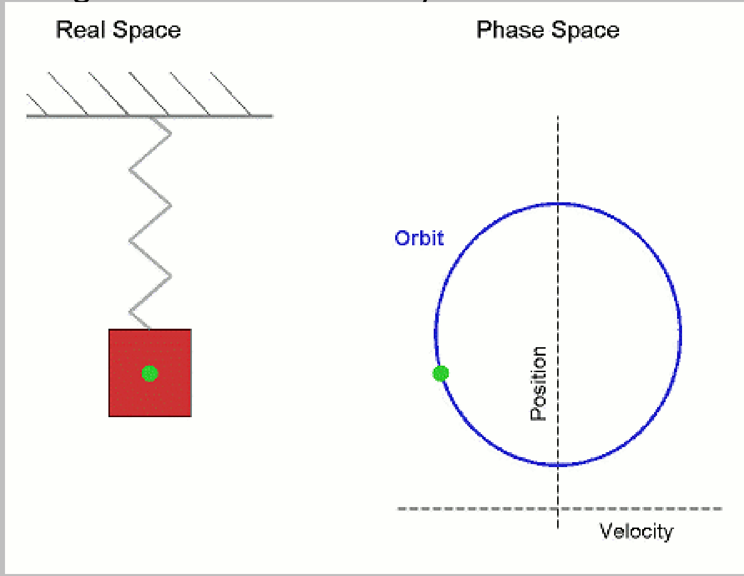


In mechanics and physics, simple harmonic motion is a special type of periodic motion where the restoring force on the moving object is directly proportional to the object's displacement magnitude and acts towards the object's equilibrium position. It results in an oscillation which, if uninhibited by friction or any other dissipation of energy, continues indefinitely.

Simple harmonic motion can serve as a mathematical model for a variety of motions, but is typified by the oscillation of a mass on a spring when it is subject to the linear elastic restoring force given by Hooke's law. The motion is sinusoidal in time and demonstrates a single resonant frequency. Other phenomena can be modeled by simple harmonic motion, including the motion of a simple pendulum, although for it to be an accurate model, the net force on the object at the end of the pendulum must be proportional to the displacement (and even so, it is only a good approximation when the angle of the swing is small; see small-angle approximation). Simple harmonic motion can also be used to model molecular vibration as well.

The motion of a particle moving along a straight line with an acceleration whose direction is always towards a fixed point on the line and whose magnitude is proportional to the distance from the fixed point is called simple harmonic motion [SHM].

In the diagram, a simple harmonic oscillator, consisting of a weight attached to one end of a spring, is shown. The other end of the spring is connected to a rigid support such as a wall. If the system is left at rest at the equilibrium position then there is no net force acting on the mass. However, if the mass is displaced from the equilibrium position, the spring exerts a restoring elastic force that obeys Hooke's law.



Once the mass is displaced from its equilibrium position, it experiences a net restoring force. As a result, it accelerates and starts going back to the equilibrium position. When the mass moves closer to the equilibrium position, the restoring force decreases. At the equilibrium position, the net restoring force vanishes. However, at $x = 0$, the mass has momentum because of the acceleration that the restoring force has imparted. Therefore, the mass continues past the equilibrium position, compressing the spring. A net restoring force then slows it down until its velocity reaches zero, whereupon it is accelerated back to the equilibrium position again.

As long as the system has no energy loss, the mass continues to oscillate. Thus simple harmonic motion is a type of periodic motion. Note if the real space and phase space diagram are not co-linear, the phase space motion becomes elliptical. The area enclosed depends on the amplitude and the maximum momentum.

Simple Harmonic Motion

Object (eg mass on a spring, pendulum bob) experiences restoring force F, directed towards the equilibrium position, proportional in magnitude to the displacement from equilibrium:

$$F = -kx$$

$$m\frac{d^2x}{dt^2} = -kx$$

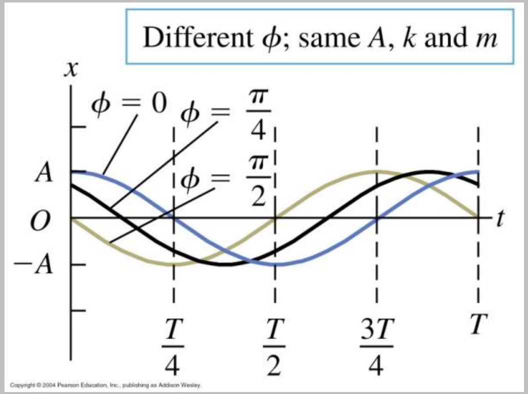
$$\frac{d^2x}{dt^2} = -\omega^2 x$$

$$\omega = \sqrt{\frac{k}{m}}$$

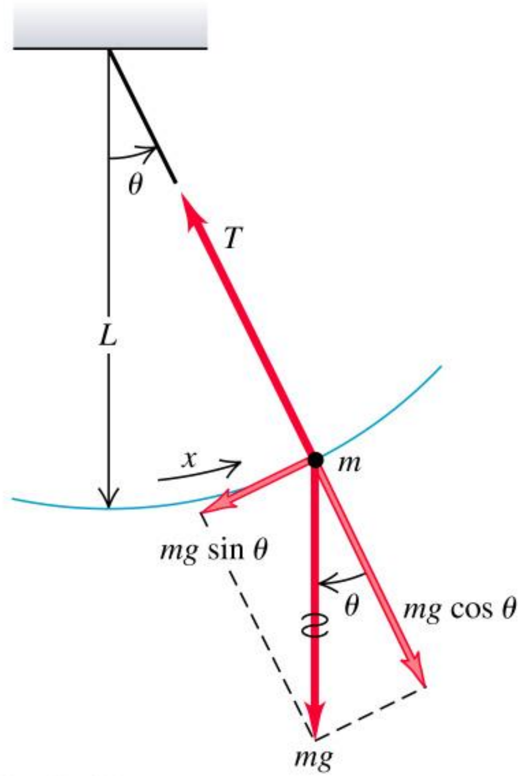
Solution:

x(t) =

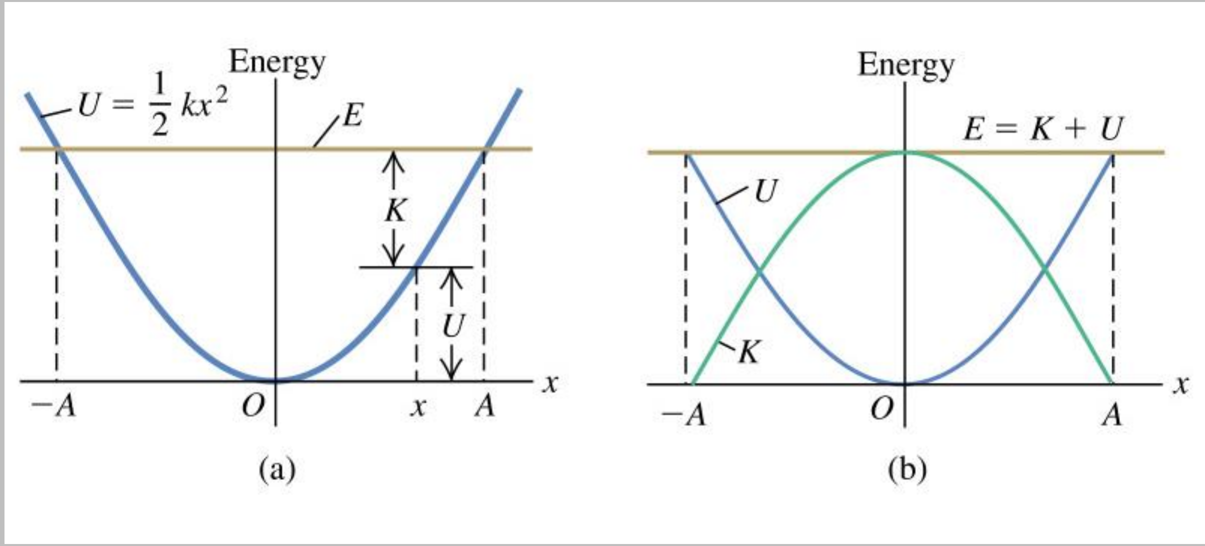
$\omega =$



Example: Simple Pendulum (small amplitude)



Energy in Simple Harmonic Motion



Damped Oscillations

In most "real life" situations, oscillations are always damped (air, fluid resistance etc)

In this case, amplitude of oscillation is not constant, but *decays* with time..(www.scar.utoronto.ca/~pat/fun/JAVA/dho/dho.html)